as Eq. (3) obtained by the first approach;

$$F_j^D e^{i\omega_0 t} = X_j^C \tag{26}$$

The force due to the segment motion without applying the assumptions used to get Eq. (18) is obtained by applying the relationship (14) to (12),

$$G_j e^{i\omega_0 t} = \zeta_j (\omega_0^2 a_{jj} - i\omega_0 b_{jj}) L$$
$$= -(a_{jj}\ddot{\zeta}_j + b_{jj}\dot{\zeta}_j) L \tag{27}$$

The coupling coefficients in Eq. (27) are zero as described previously. The force due to the motion of the segment in still water, Eq. (27) obtained by the potential flow theory is exactly the same as Eq. (4) obtained by the first approach

$$G_j e^{i\omega_0 t} = X_j^M \tag{28}$$

In summary, the two approaches give idential equations of the hydrodynamic forces on the platform segment in the (x,y,z) coordinate with the assumptions made to get Eq. (18). Without these assumptions, the second approach[8] is more general than the first approach. However, the computed results by the two approaches for a floating platform in waves in the $(\bar{x},\bar{y},\bar{z})$ coordinate system can be compared with caution. There are the hydrodynamic interferences at the joints of the vertical and horizontal segments and between the adjacent segments. The hydrodynamic interferences at the joint can generate a three-dimensional flowfield. For the three-dimensional hydrodynamic interferences at the joints, the equations of the diffraction force and motion-dependent force should use the added mass and damping coefficients which take into account the hydrodynamic interferences at the joints: if the interferences are strong, theory may not be valid. The first approach, within the framework of the theory[9] can be compared to the second approach with the computed results which take into account the hydrodynamic interferences. For this reason the computed results by Kim[8] can be compared to the first approach only with caution. It is noted that both approaches have been applied to many platform problems with reasonably good accuracy of the computed results.

References

[1] Bain, J. A., "Extension of MOHOLE Platform Force and Motion Studies," Oct. 1964, Project MOHOLE Report by General Electric Co.

[2] Burke, B. G., "The Analysis of Motions of Semisubmersible Drilling Vessels in Waves," *Proceedings of Offshore Technology Conference*, Houston, Texas, 1969, Paper 1024.

[3] Paulling, J. R., "Elastic Response of Stable Platform Structures to Wave Loading," *Proceedings of the International Symposium on the Dynamics of Marine Vehicles and Structures in Waves*, Institute of Mechanical Engineers, London, England, April 1974, pp. 263-272.

[4] Ochi, M. K. and Vuolo, R. M., "Seakeeping Characteristics of a Multi-Unit Ocean Platform," *Transactions of the Society of Naval Architects and Marine Engineers*, Honolulu, Hawaii, May 1971.

[5] Hooft, J. P., "A Mathematical Method of Determining Hydrodynamically Induced Forces on a Semisubmersible," *Transactions of the Society of Naval Architects and Marine Engineers*, Vol. 79, New York, 1971, pp. 28-70.

[6] Chung, J. S., "Motions of Semisubmersible Drilling Rigs in Deep Water," *Journal of the Society of Naval Architects of Korea*, Vol. 11, Nov. 1974, pp. 23-40.

[7] Wiegel, R. L., *Oceanographical Engineering*, Prentice-Hall, Englewood Cliffs, N. J., 1964, pp. 254.

[8] Kim, C. H. and Chou, F., "Motions of a Semi-Submersible Drilling Platform in Head Seas," OE 71-8, Dec. 1971, Ocean Engineering Department, Stevens Institute of Technology, Hoboken, N. J.

[9] Salveson, N., Tuck, E. O., and Faltinson, O., "Ship Motions and Loads," *Transactions of the Society of Naval Architects and Marine Engineers*, Vol. 78, New York, 1970, pp. 250-287.

Turbulent Wake of an Axisymmetric, Self-Propelled Body

M. Robinson Swift*
University of New Hampshire, Durham, N. H.

Nomenclature

A,B,C,C_1,C_2,D	= constants
ℓ	= length scale
u'_m	= turbulent velocity scale
u'_{max}	= measured maximum turbulent intensity at a given axial position
U_0	= freestream velocity
$-\rho\langle u'_r u'_z\rangle$	= Reynolds stress
U_d	= velocity defect scale
U_z	= mean axial velocity
$r_{1/2}$	= measured length scale
r,z	= radial, axial coordinate
ϵ	= kinematic eddy viscosity

Introduction

THOUGH free turbulent shear flows have been investigated extensively, both analytically and experimentally, comparatively little analytical work has been done in studying the wakes of bodies with hydrodynamic self-propulsion.[1] A very complete experimental study was done, however, in which the wake of a totally immersed, axisymmetric, self-propelled body was simulated in an air tunnel using a concentric nozzle and disk. The results, reported by Naudascher,[2] provide enough data to check the validity of proposed hypotheses of the wake behavior and also the corresponding analytical solutions. One important conclusion from the data was that various flow characteristics, such as mean velocity, turbulence intensity and turbulent Reynolds stress, attained a self-preserving form. Thus, a self-preservation hypothesis seems justified as the basis for an analytical approach to the problem. That is, the flow characteristics may be assumed invariant along the wake axis if expressed in terms of appropriate length and velocity scales. Similarity solutions of this type are given by Birkhoff and Zarantonello,[3] and Tennekes and Lumley[4] in which one length and one velocity scale are used. The results indicate that the length scale should have the form $C_1 z^{1/5}$, while the velocity scale should vary as $C_2 z^{-4/5}$. No analytical expression, however, was given for the corresponding mean velocity profile in the axisymmetric case.

When these classical results are compared with Naudascher's data, plotted in Fig. 1, it is seen that the predicted axial dependence of the length scale is roughly the same as the data. The expression for the velocity scale, on the other hand, agrees well with the observed data for the turbulence intensity, but not with the mean velocity defect. In fact, as pointed out by Schetz and Favin,[1] Naudascher's data show that the turbulence intensity and the velocity defect do not vary axially in the same way, thus distinguishing this case from more conventional wake flows.

In this Note, therefore, a new similarity solution is developed in which two velocity scales are used—one for the mean velocity defect and one for turbulent velocity fluctuations. Expressions for the mean velocity profile as well as the velocity and length scales are developed.

Received February 24, 1975; revision received June 9, 1975.
Index category: Jets, Wakes, and Viscid-Inviscid Flow Interactions.
*Postdoctoral Research Associate, Mechanics Research Lab.

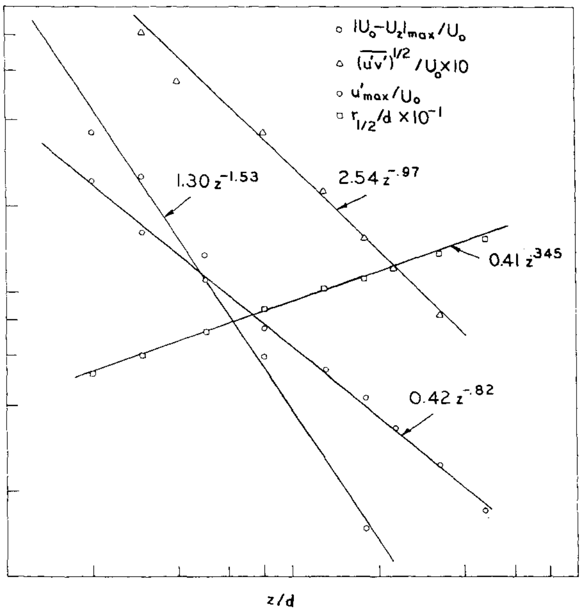

Fig. 1 Length and velocity scales from Naudascher.

Proposed Similarity Solution

Using boundary-layer approximations modified for wake flows with slow lateral spreading, the momentum equation may be written as[5]

$$U_0(\partial U_z/\partial z) + 1/r(\partial/\partial r)r<u_r'u_z'> = 0 \tag{1}$$

Next a similarity or self-preservation hypothesis for the wake structure and an eddy viscosity model for the turbulent shear will be employed in the following manner:

$$(U_0 - U_z/U_d) = f(r/\ell) = f(\xi), -<u_r'u_z'> = \epsilon(\partial U_z/\partial r), \epsilon = C\ell u_m' \tag{2}$$

where

$$U_d = U_d(z) = |U_0 - U_z|_{\max} = \text{velocity defect scale}$$

$$\ell = \ell\ (z) = \text{length scale}$$

and

$$u_m' = u_m'(z) = \text{turbulent velocity fluctuation scale} \tag{3}$$

The velocity scale U_d characterizes the evolution of the velocity defect along the wake axis, while u_m' is appropriate to the random, small-scale fluid motion. Based on Naudascher's data, an empirical relation between U_d and u_m' may be approximated by

$$u_m' = D(U_0 U_d)^{1/2} \tag{4}$$

It has been assumed that the turbulent eddy viscosity depends on the turbulent fluctuations, rather than on either freestream velocity or the mean velocity defect.

Substituting the expression for the turbulent shear from Eq. (2) into Eq. (1), multiplying by r^3, and integrating with respect to r will give

$$\frac{\partial}{\partial z}\int_0^\infty r^3 U_0(U_z - U_0)dr = \epsilon\int_0^\infty r^2 \frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}(U_z - U_0)\right]dr = \frac{2\epsilon}{\pi}\int_0^\infty (U_z - U_0)\ (2\pi r dr) = 0 \tag{5}$$

if the conservation of mass flow is taken into account. In terms of the self-preservation forms of the solution given by Eqs. (2) and (3), Eq. (5) becomes

$$(U_d\ell^4)\int_0^\infty \xi^3 f(\xi)d\xi = \text{constant} \tag{6}$$

The simplified equation of motion, written in terms of the self-preservation solution and the eddy viscosity model given by Eq. (2), becomes

$$\left(\frac{\ell U_0}{Cu_m'U_d}\frac{dU_d}{dz}\right)f - \left(\frac{U_0}{Cu_m'}\frac{d\ell}{dz}\right)\xi\frac{df}{d\xi} + 1/\xi\frac{df}{d\xi} + \frac{d^2f}{d\xi^2} = 0 \tag{7}$$

For a similarity solution to exist, the coefficients in parentheses in the last two equations must be constants. This requirement and Eq. (4) are satisfied by

$$U_d = Az^{-4/3}, u_m' = D(U_0A)^{1/2}z^{-2/3}, \ell = Bz^{1/3} \tag{8}$$

where A, B, and D are constants.

Substituting the previous expressions for the length and velocity scales into Eq. (7) will give

$$\left[\frac{1/3\,U_0B}{CD(U_0A)^{1/2}}\right]\left(4f + \xi\frac{df}{d\xi}\right) + \frac{1}{\xi}\frac{df}{d\xi} + \frac{d^2f}{d\xi^2} = 0 \tag{9}$$

The constant A is determined by Eq. (3), while D must be such that Eq. (4) is satisfied. The constant B, however, is still arbitrary. The constant B is made definite by requiring that the term in brackets in Eq. (9) has magnitude unity. The resulting differential equation,

$$(d^2f/d\xi^2) + (1/\xi + \xi)(df/d\xi) + 4f = 0 \tag{10}$$

has a solution

$$f = (1/2\xi^2 - 1)e^{-1/2\xi^2} \tag{11}$$

so that the mean axial velocity profile is given by

$$U_z = U_0 + U_d[1 - 1/2(r/\ell)^2]e^{-1/2(r/\ell)^2} \tag{12}$$

Discussion

Since the available data for the wake structure of a self-propelled body indicate that the mean velocity defect and the turbulent velocity fluctuations do not decay along the axis at the same rate, the classical approach to obtaining similarity solutions using one velocity scale is not adequate. The new similarity solution previously given, which uses two empirically related velocity scales, can be more satisfactorily compared with the experimental results. Though the predicted decay rates for the velocity scales are somewhat low, the analytical results for the length scale ℓ and for the turbulent shear,

$$<u_r'u_z'> = \left(ACD(U_0A)^{1/2}\frac{df}{d\xi}\right)z^{-2} \tag{13}$$

are in excellent agreement with the data.

It is interesting to note that the turbulent velocity scale and the characteristic length scale for the self-propelled wake are similar to, respectively, the velocity and length scales for a finite momentum, axisymmetric wake.[3,4] The axial dependence of the velocity defect scale, on the other hand, is observed to be nearly proportional to the square of the turbulent velocity scale.

References

[1] Schetz, J. A. and Favin, S., "Analysis of Free Turbulent Mixing Flows without a Net Momentum Defect," *AIAA Journal*, Vol. 10, Nov. 1972, pp. 1524-1526.

[2] Naudascher, E., "Flow in the Wake of Self-Propelled Bodies and Related Sources of Turbulence," *Journal of Fluid Mechanics*, Vol. 22, Pt. 4, 1965, pp. 625-656.

[3] Birkhoff, G. and Zarantonello, E. H., *Jets, Wakes, and Cavities*, Academic Press, New York, 1957, pp. 307-309.

[4] Tennekes, H. and Lumley, J. L., *A First Course in Turbulence*, MIT Press, Cambridge, Mass., 1972, p. 134.

[5] Monin, A. S. and Yaglom, A. M., *Statistical Fluid Mechanics*, MIT Press, Cambridge, Mass., 1971, p. 356.

Index of Papers Published in Journal of Hydronautics

Volume 9—January through October 1975

Issue	Pages
January	1-48
April	49-80
July	81-128
October	129-176

SYN, SA, EN, TC, and ERR indicate Synoptic, Survey Article, Engineering Note, Technical Comment, and Errata, respectively. All other items are regular full-length articles. In the Subject Index, papers are listed alphabetically by title, regardless of category.

SUBJECTS

AIRCRAFT TECHNOLOGY, VTOL

Ground- (or Water-Surface) Effect Machines

FLUID DYNAMICS

Boundary Layers and Convective Heat Transfer – Laminar

Boundary Layers and Convective Heat Transfer – Turbulent

Hydrodynamics

Jets, Wakes, and Viscid-Inviscid Flow Interactions

Multiphase Flows

Nozzle and Channel Flow

Viscous Nonboundary-Layer Flows